Lecture 11

More Examples on Mathematical Induction, Flawed Proofs

Tiling Using L-shape Tile

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Theorem: $\forall n \in \mathbb{Z}^+$, there exists a tiling of a $2^n \times 2^n$ courtyard

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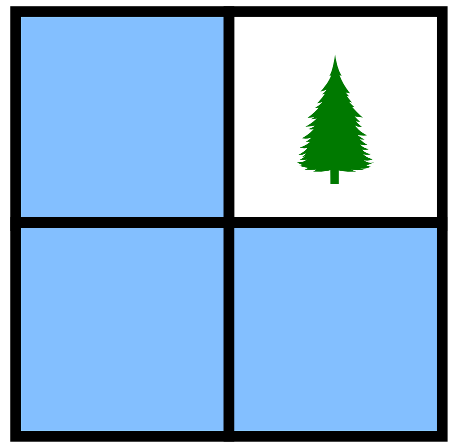

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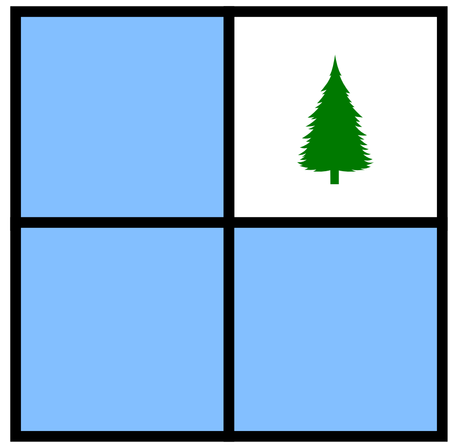

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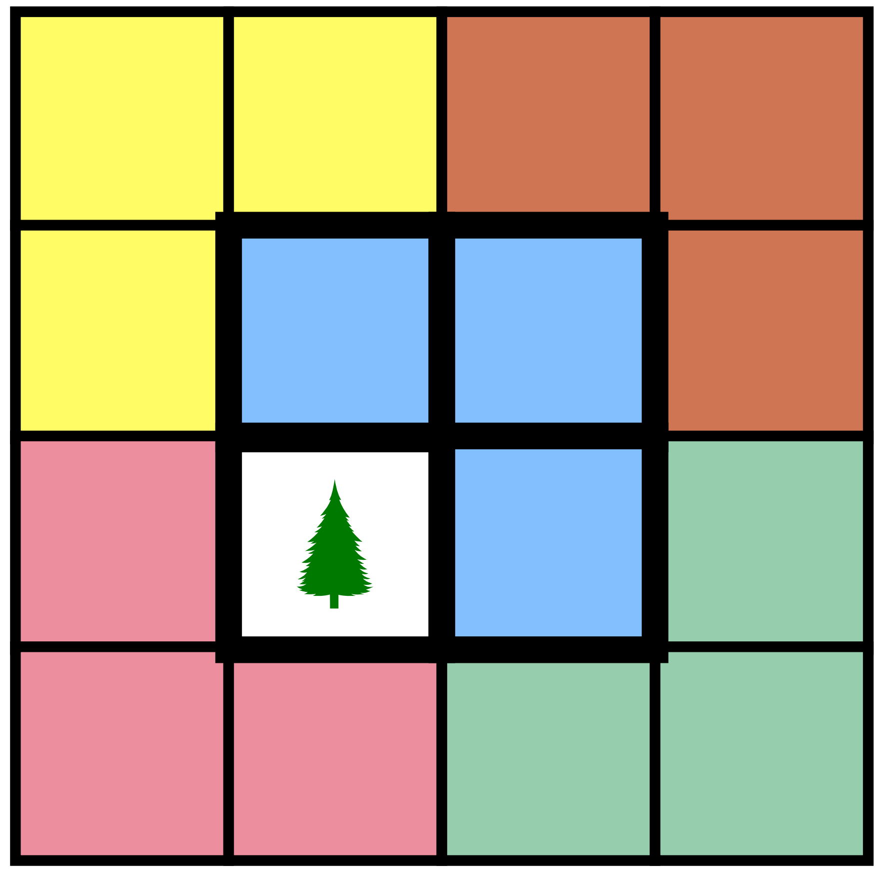

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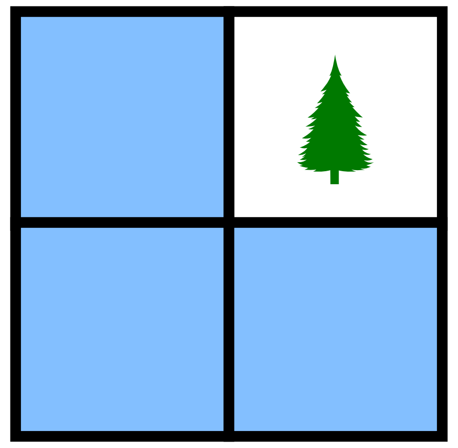

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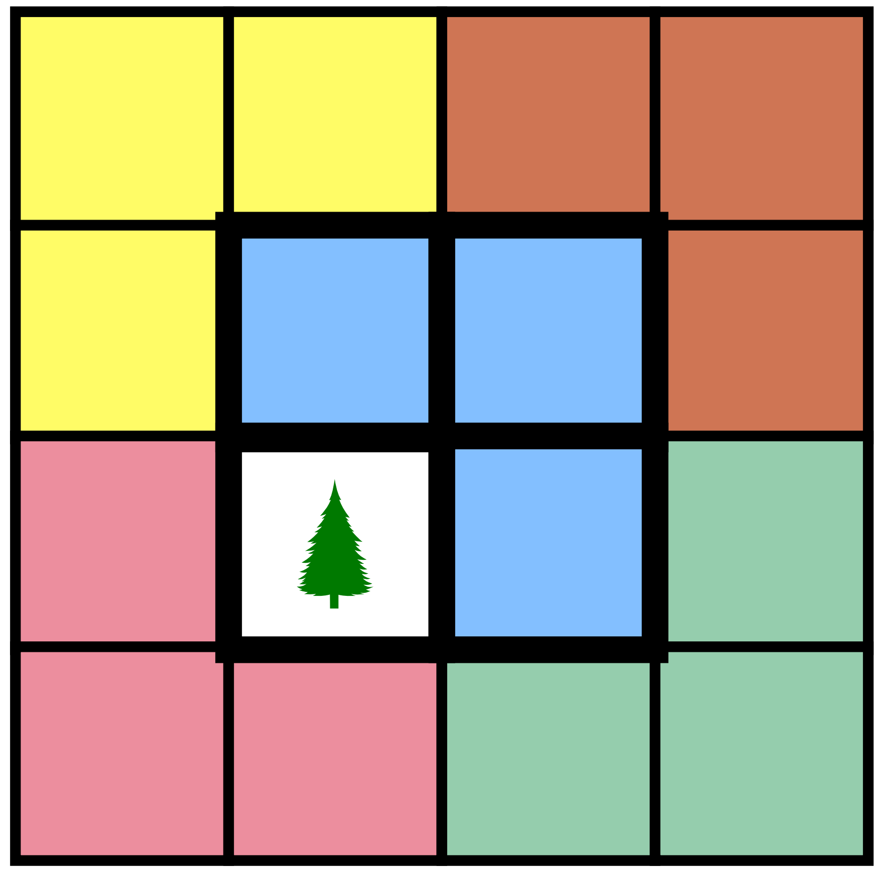

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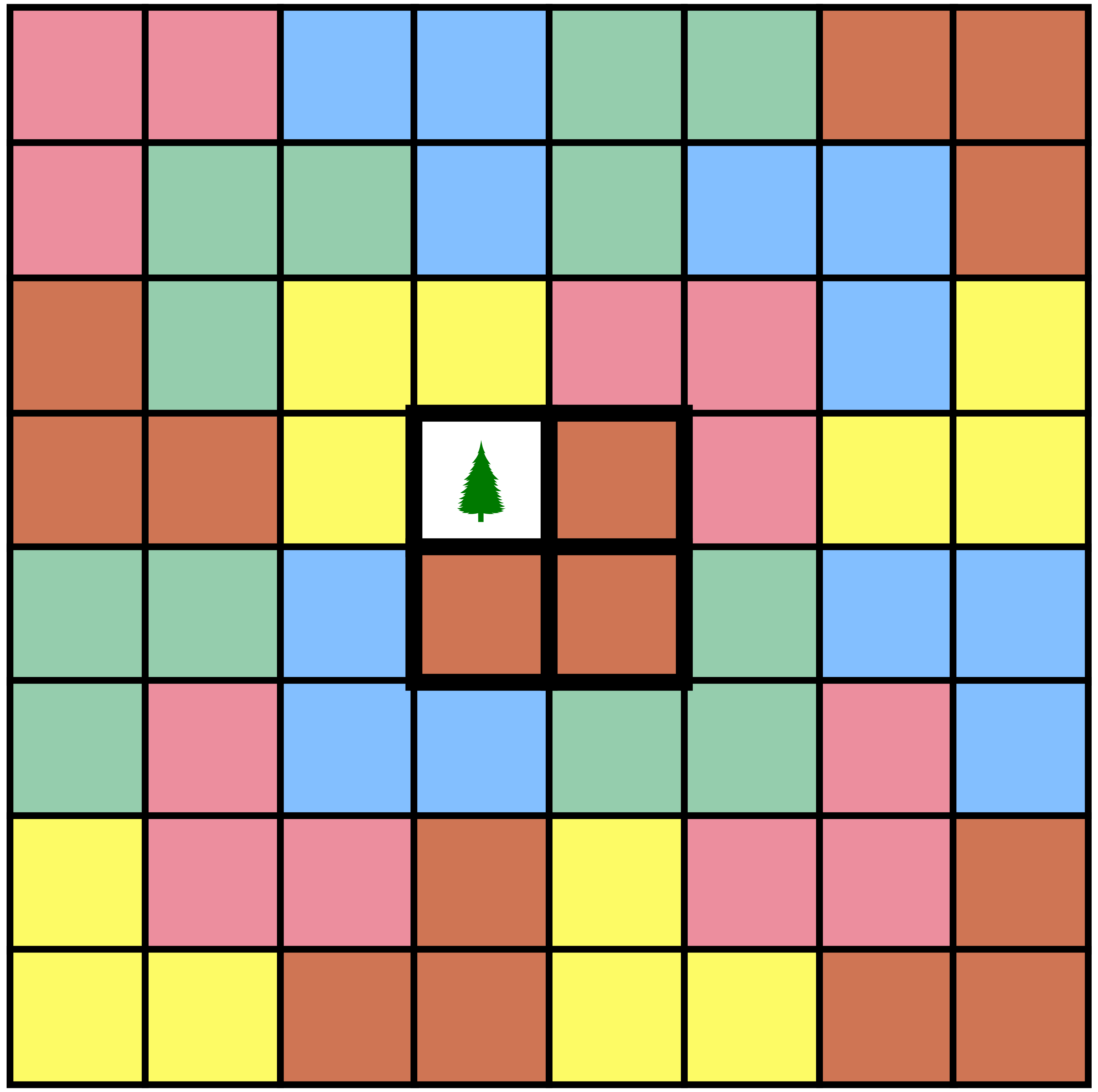

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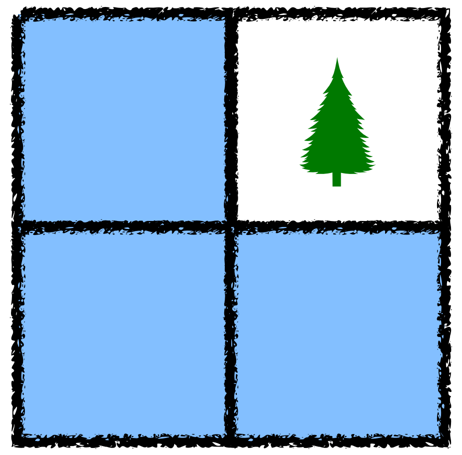

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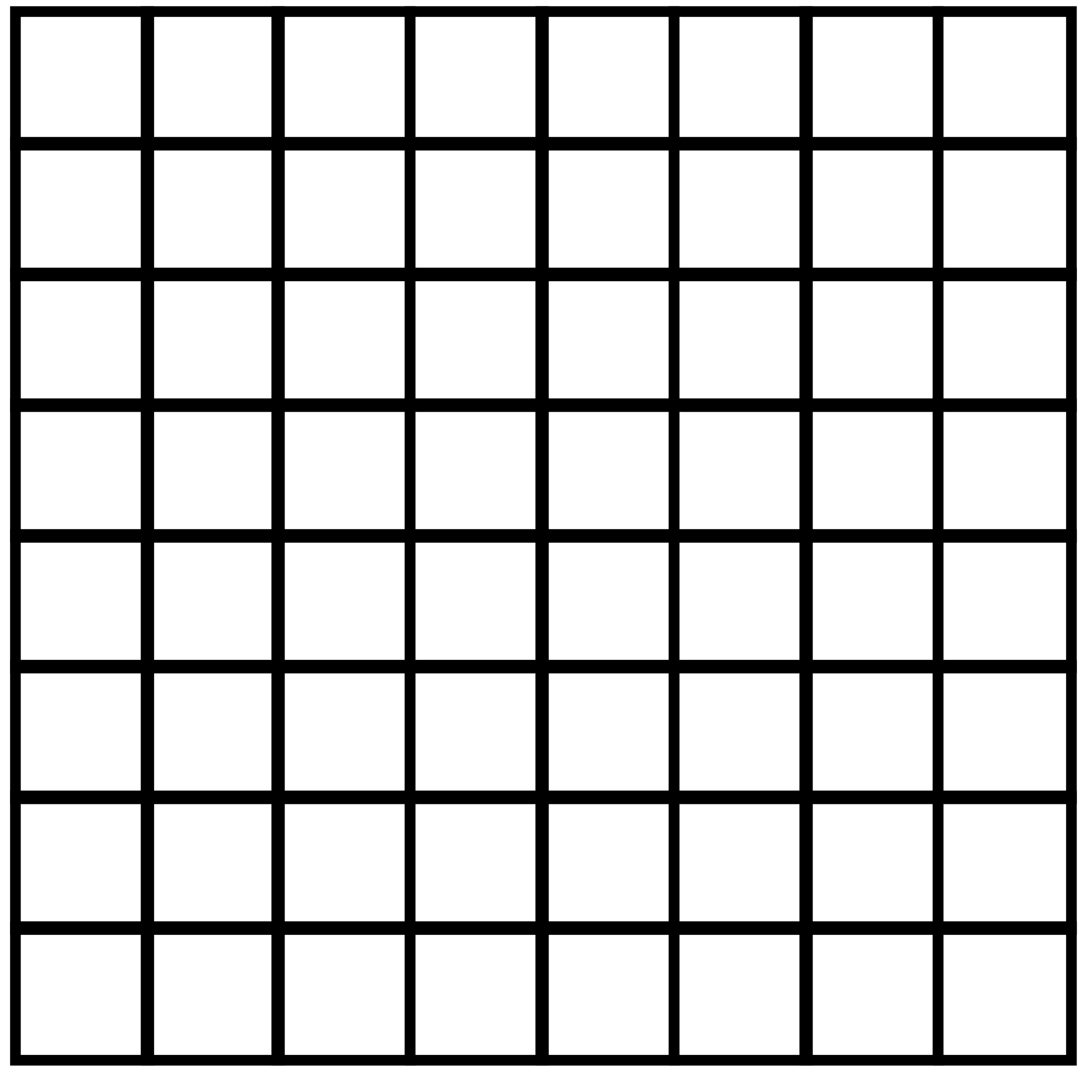

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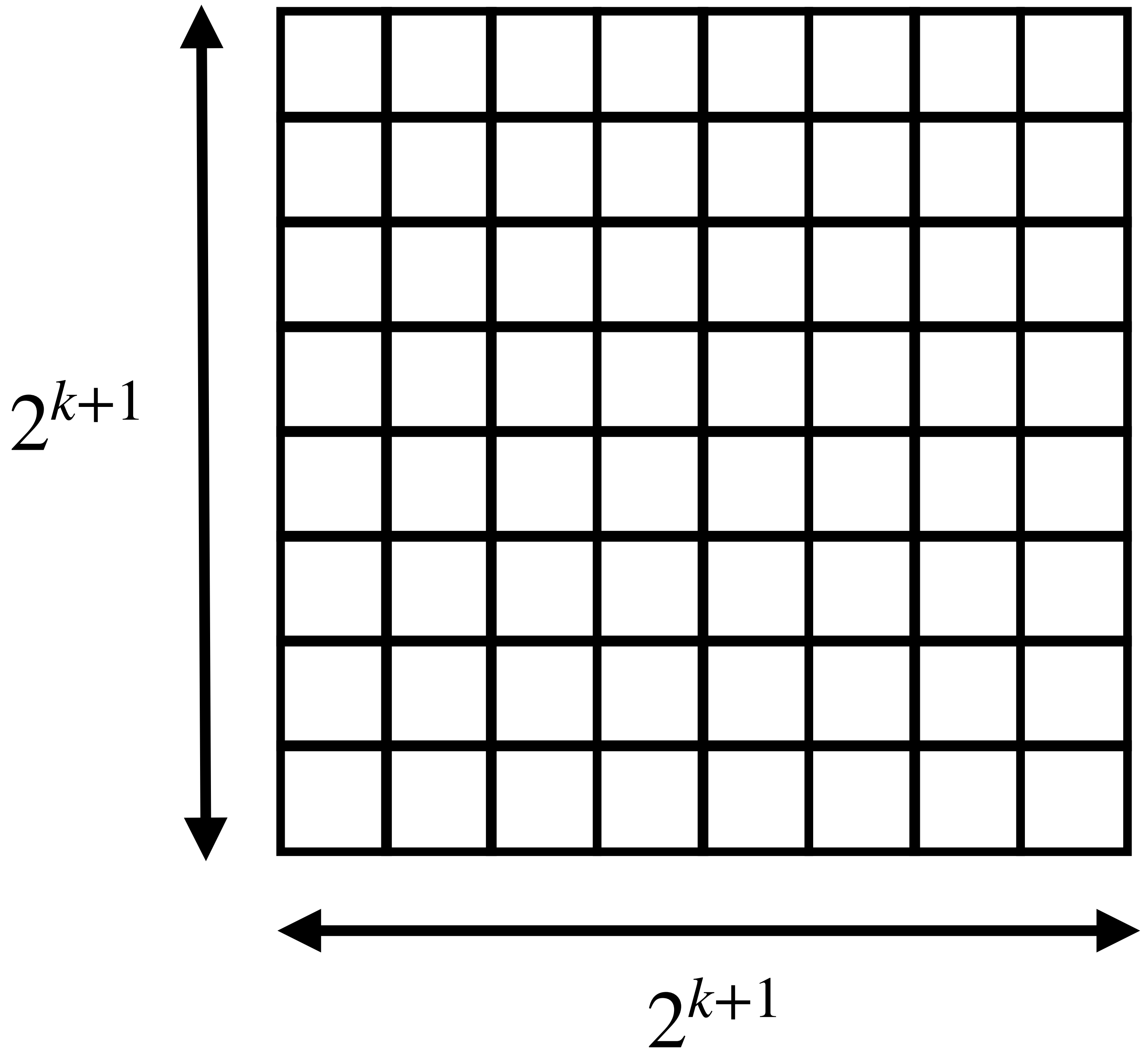

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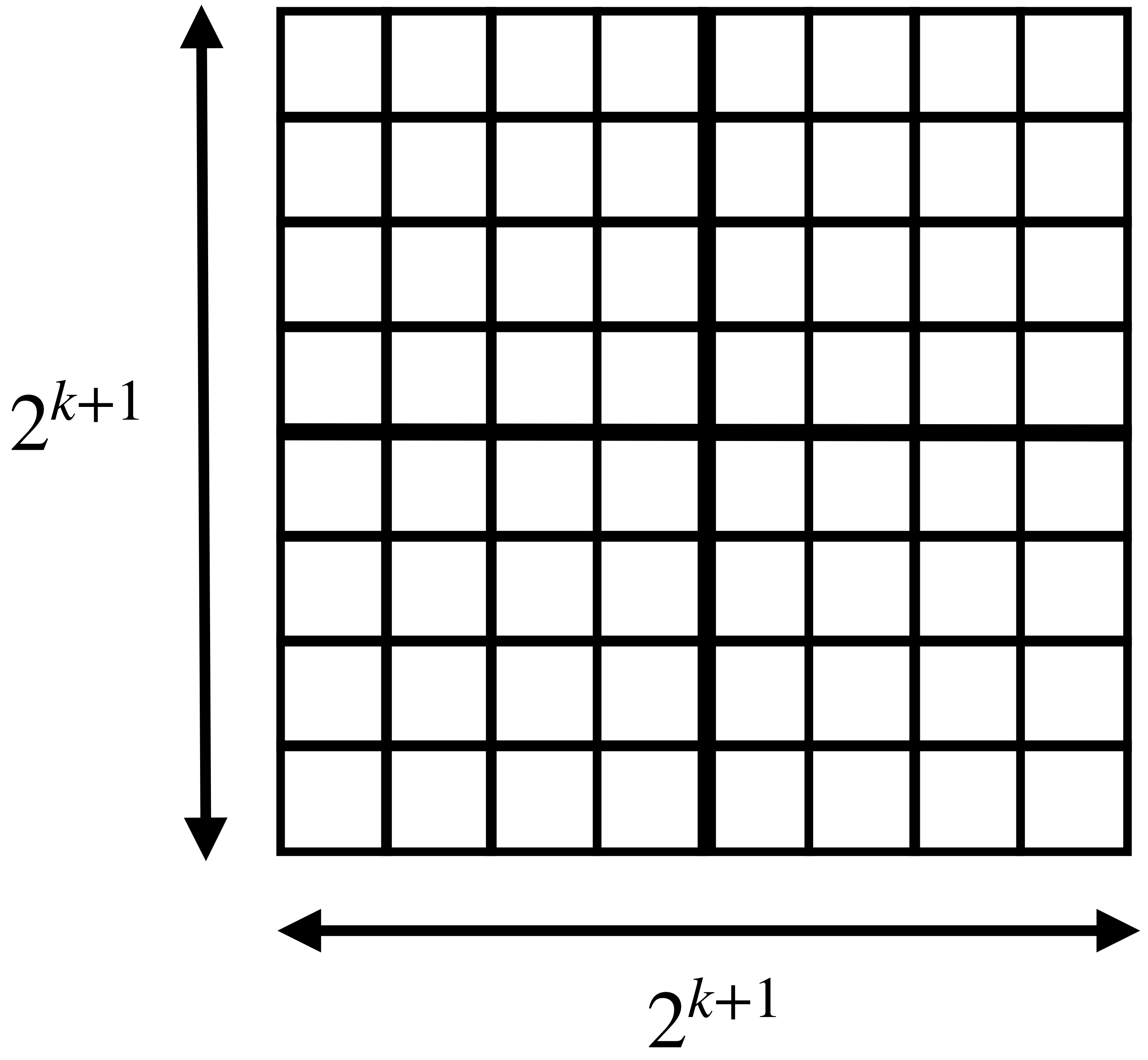

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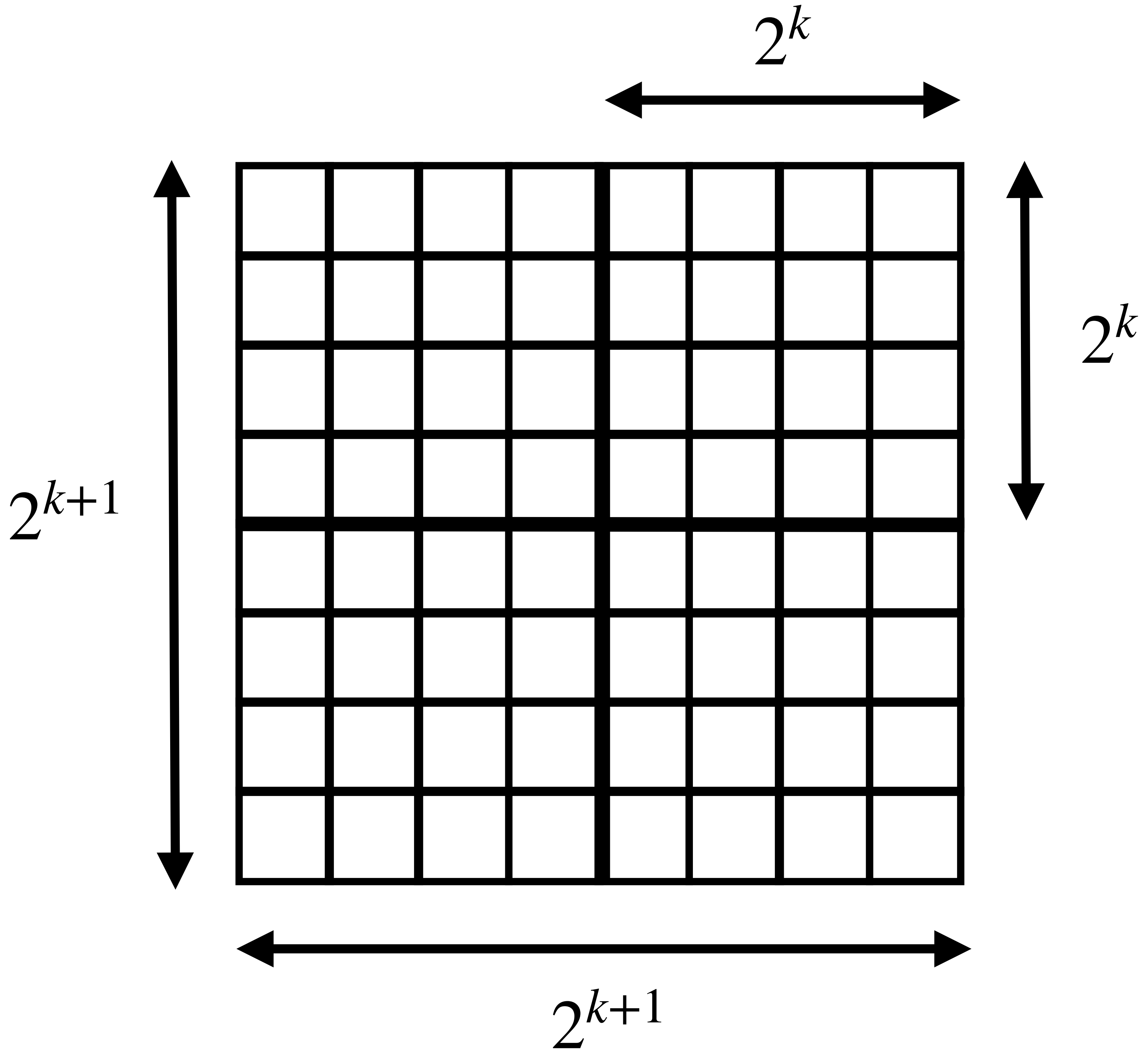

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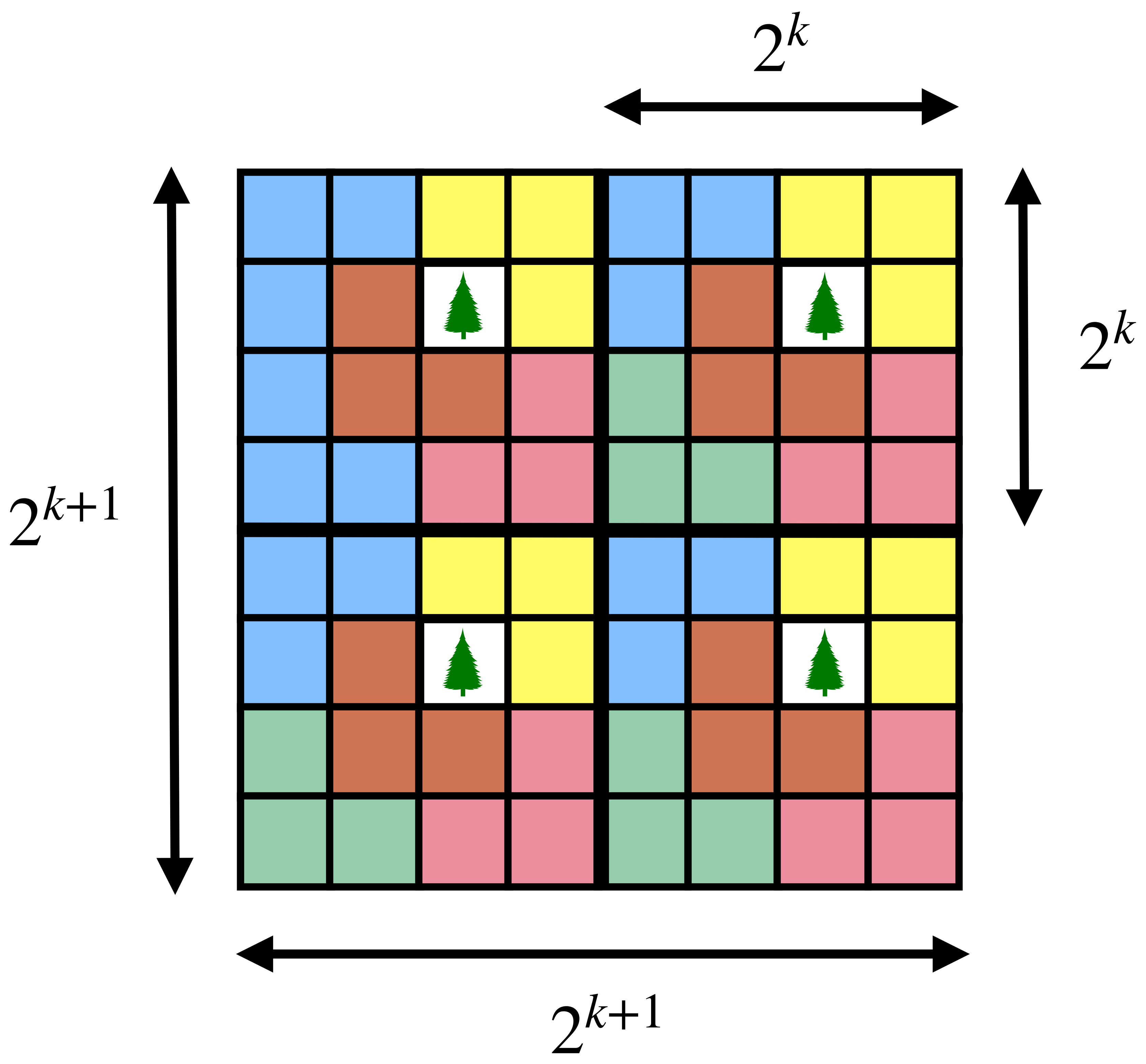

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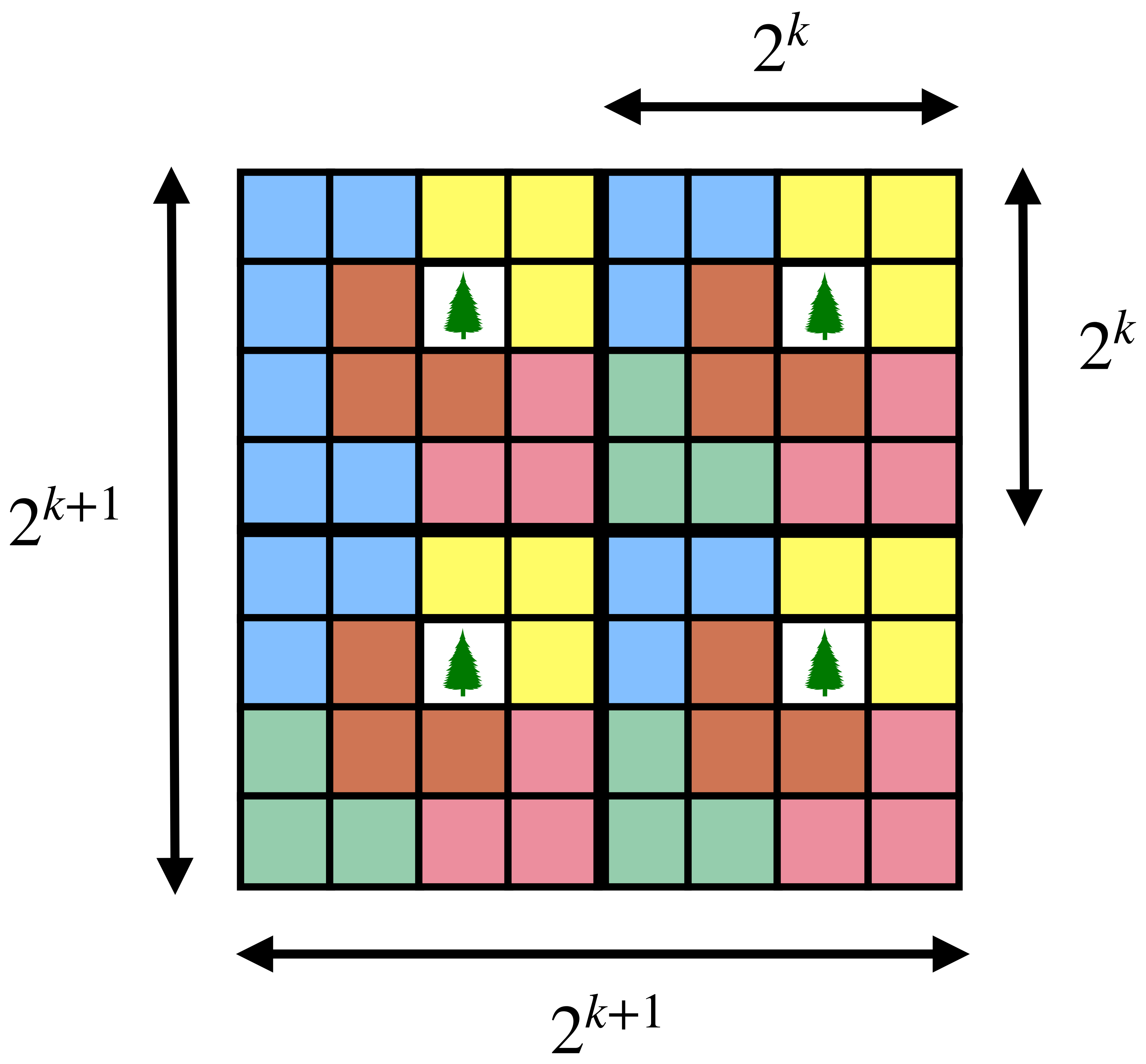

How to proceed from here?

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Previous Theorem: $\forall n \in \mathbb{Z}^+$, there exists a tiling of a $2^n \times 2^n$ courtyard using L-shaped tiles with

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Tip: Sometimes stronger statement is easier to prove using induction.

Let's create a stronger version!

Previous Theorem: $\forall n \in \mathbb{Z}^+$, there exists a tiling of a $2^n \times 2^n$ courtyard using L-shaped tiles with a tree in a central square.

Tiling Using L-shape Tile

Tip: Sometimes stronger statement is easier to prove using induction.

Let's create a stronger version!

Previous Theorem: $\forall n \in \mathbb{Z}^+$, there exists a tiling of a $2^n \times 2^n$ courtyard using L-shaped tiles with a tree in a central square.

Stronger Theorem: $\forall n \in \mathbb{Z}^+$, there exists a tiling of a $2^n \times 2^n$ courtyard using L-shaped tiles with

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Tip: Sometimes stronger statement is easier to prove using induction.

Let's create a stronger version!

Previous Theorem: $\forall n \in \mathbb{Z}^+$, there exists a tiling of a $2^n \times 2^n$ courtyard using L-shaped tiles with a tree in a central square.

Stronger Theorem: $\forall n \in \mathbb{Z}^+$, there exists a tiling of a $2^n \times 2^n$ courtyard using L-shaped tiles with a tree in any square.

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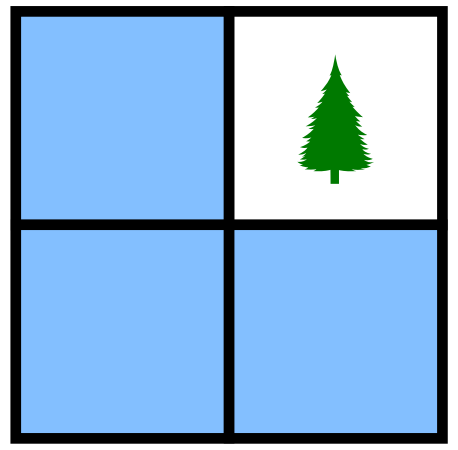

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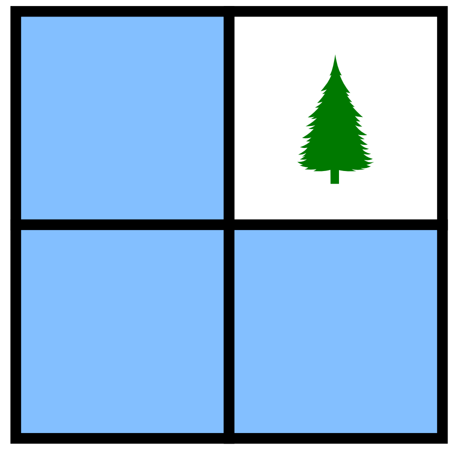

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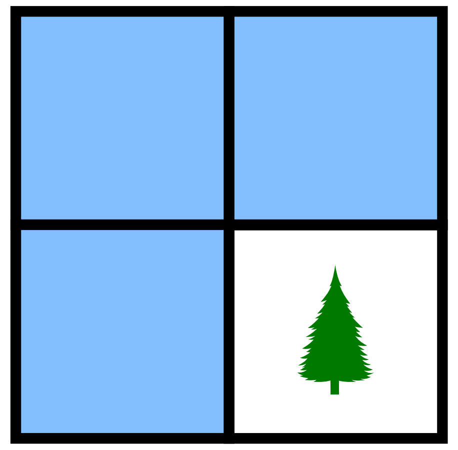

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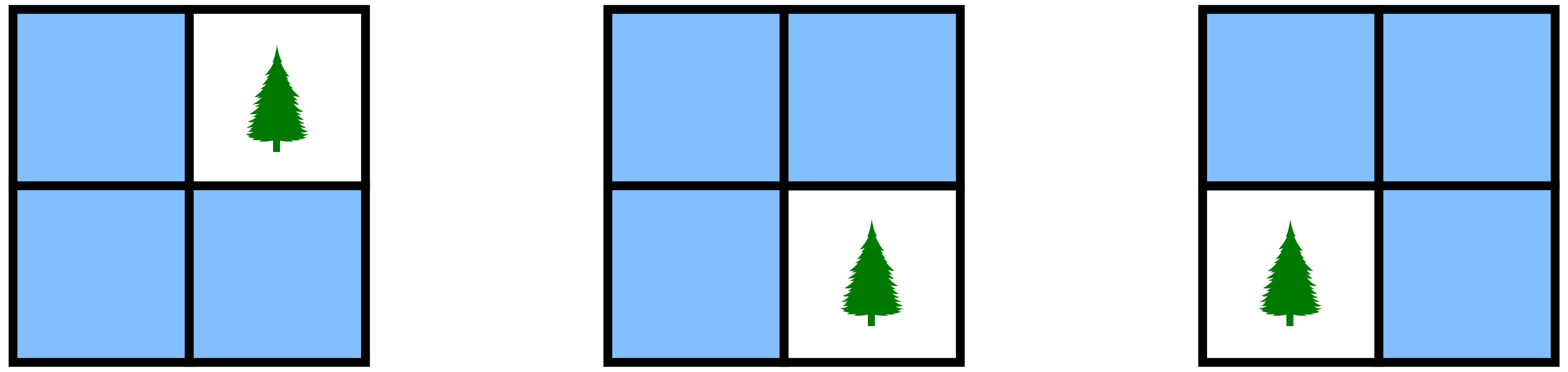

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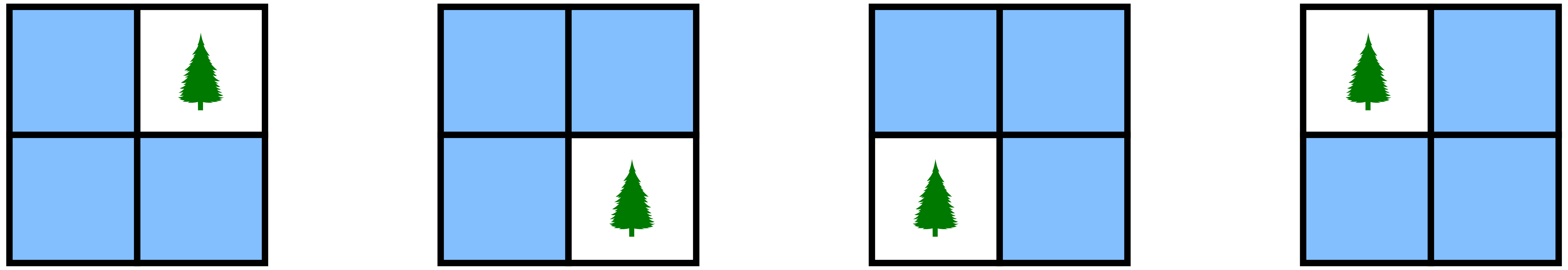

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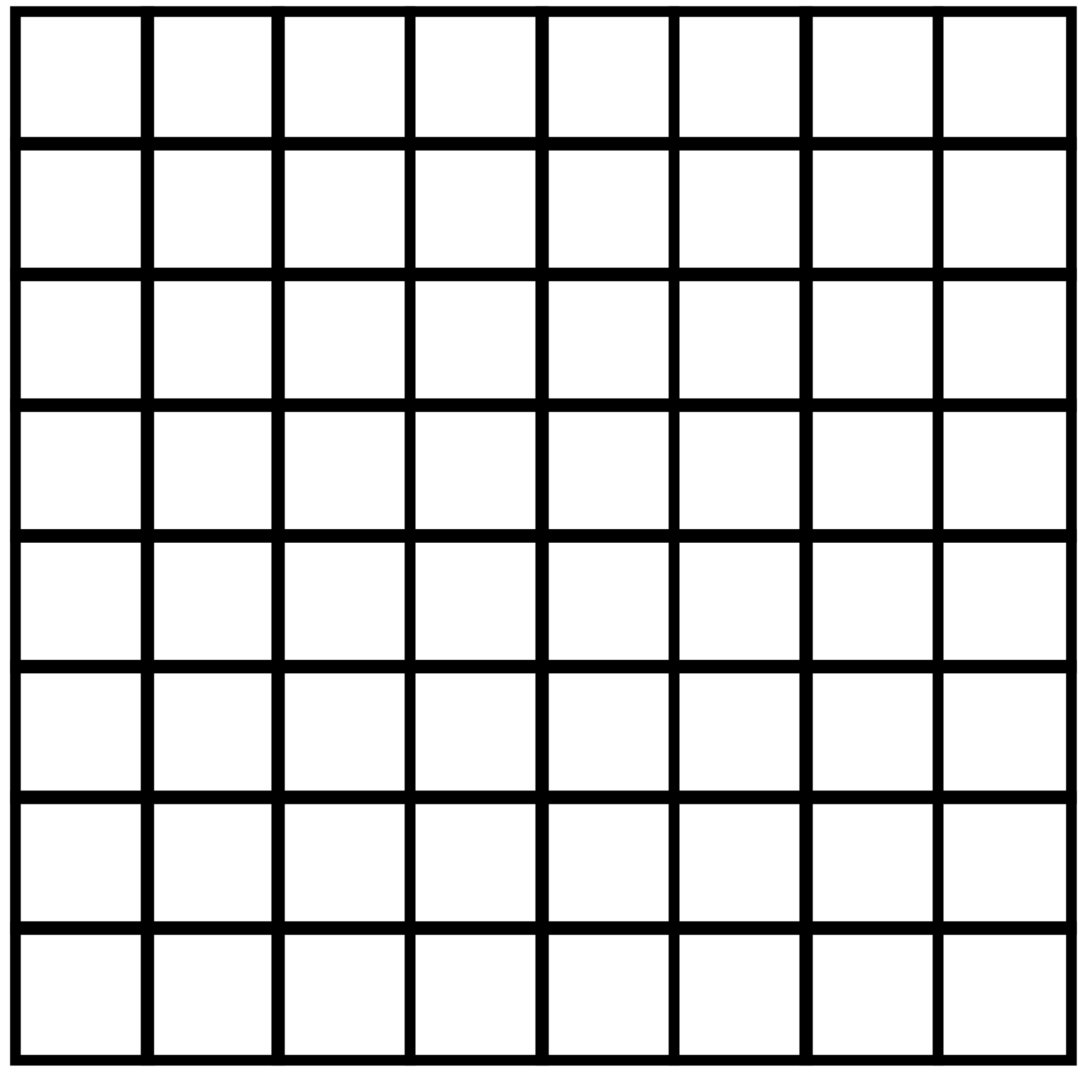

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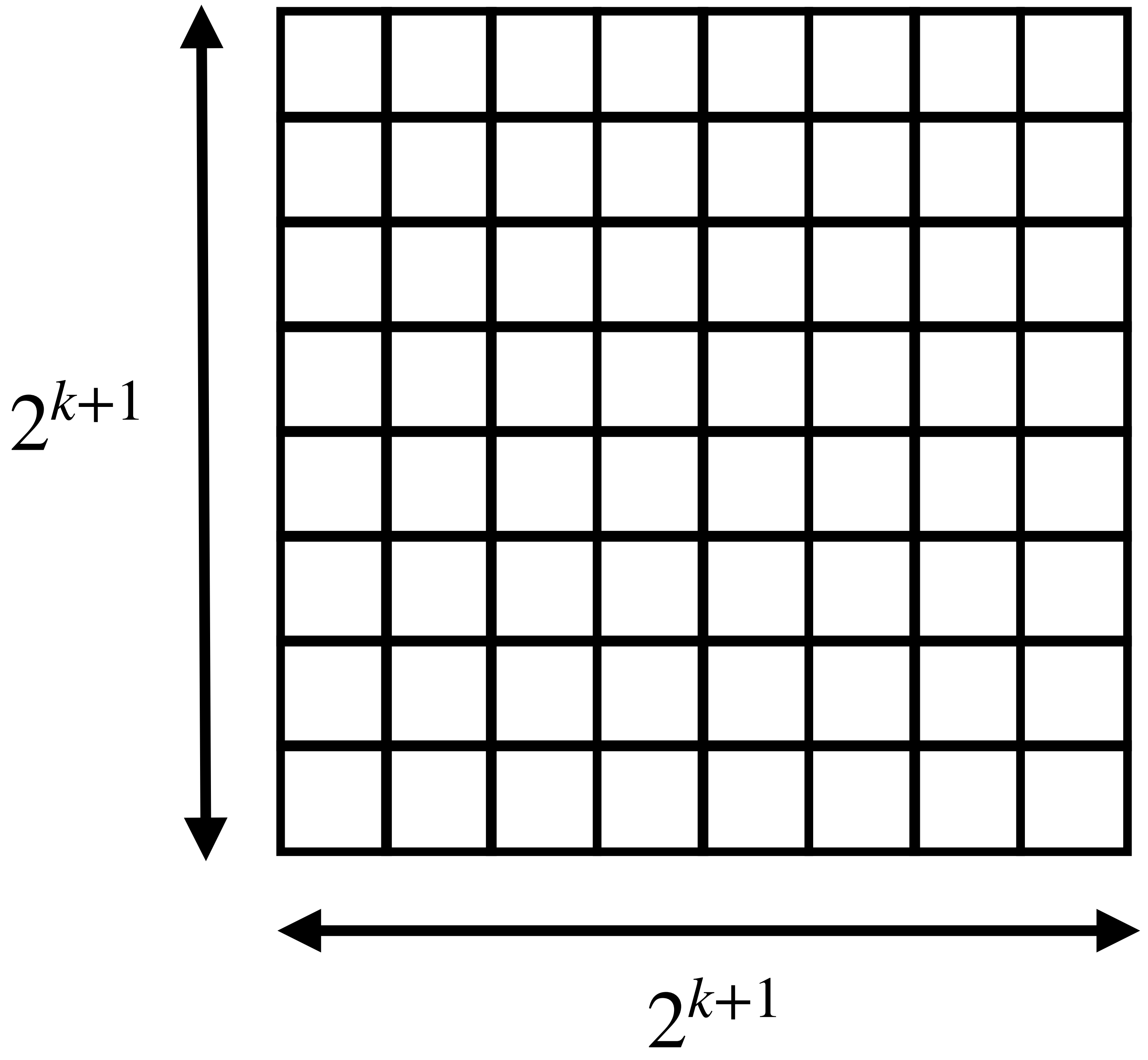

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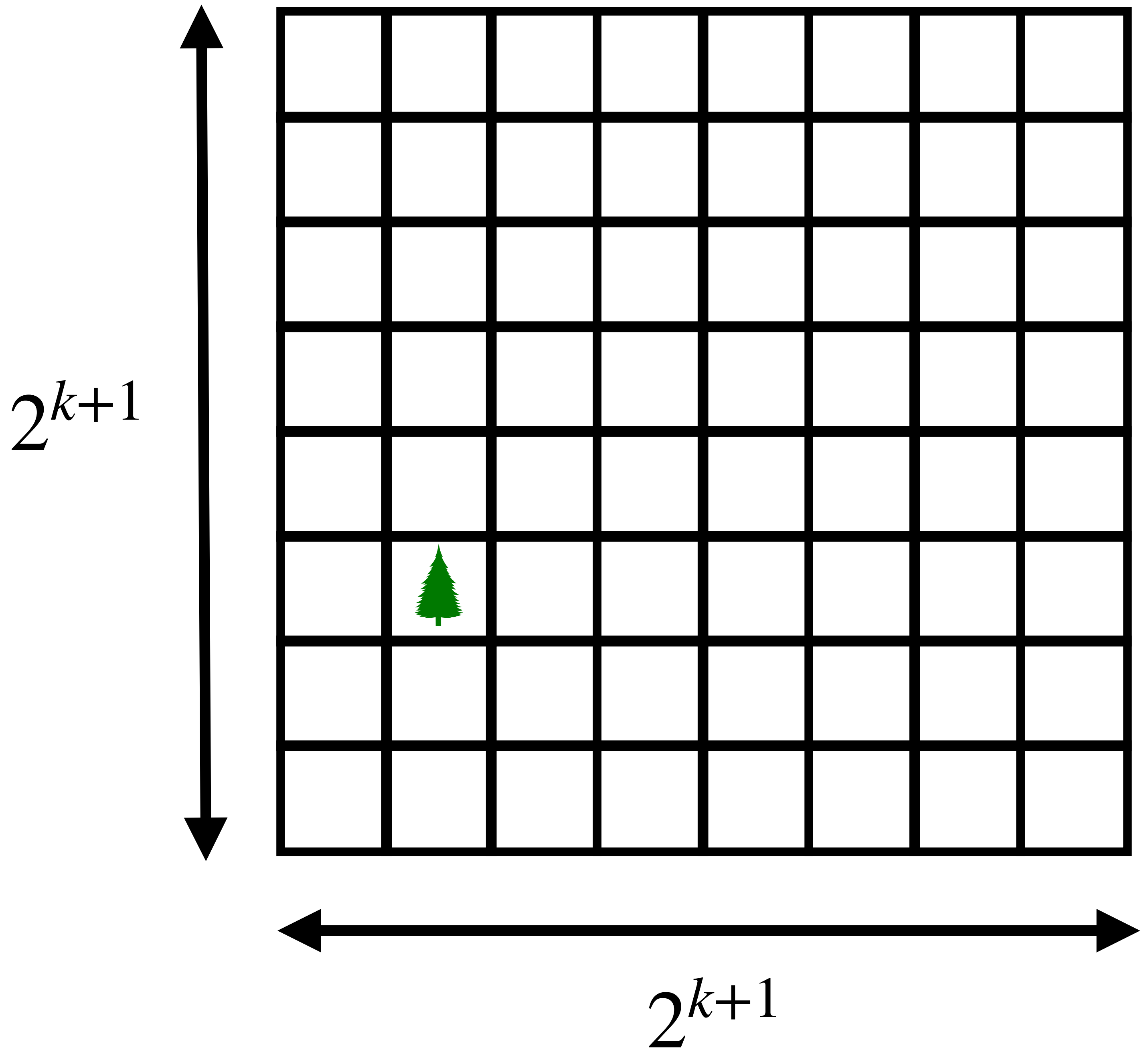

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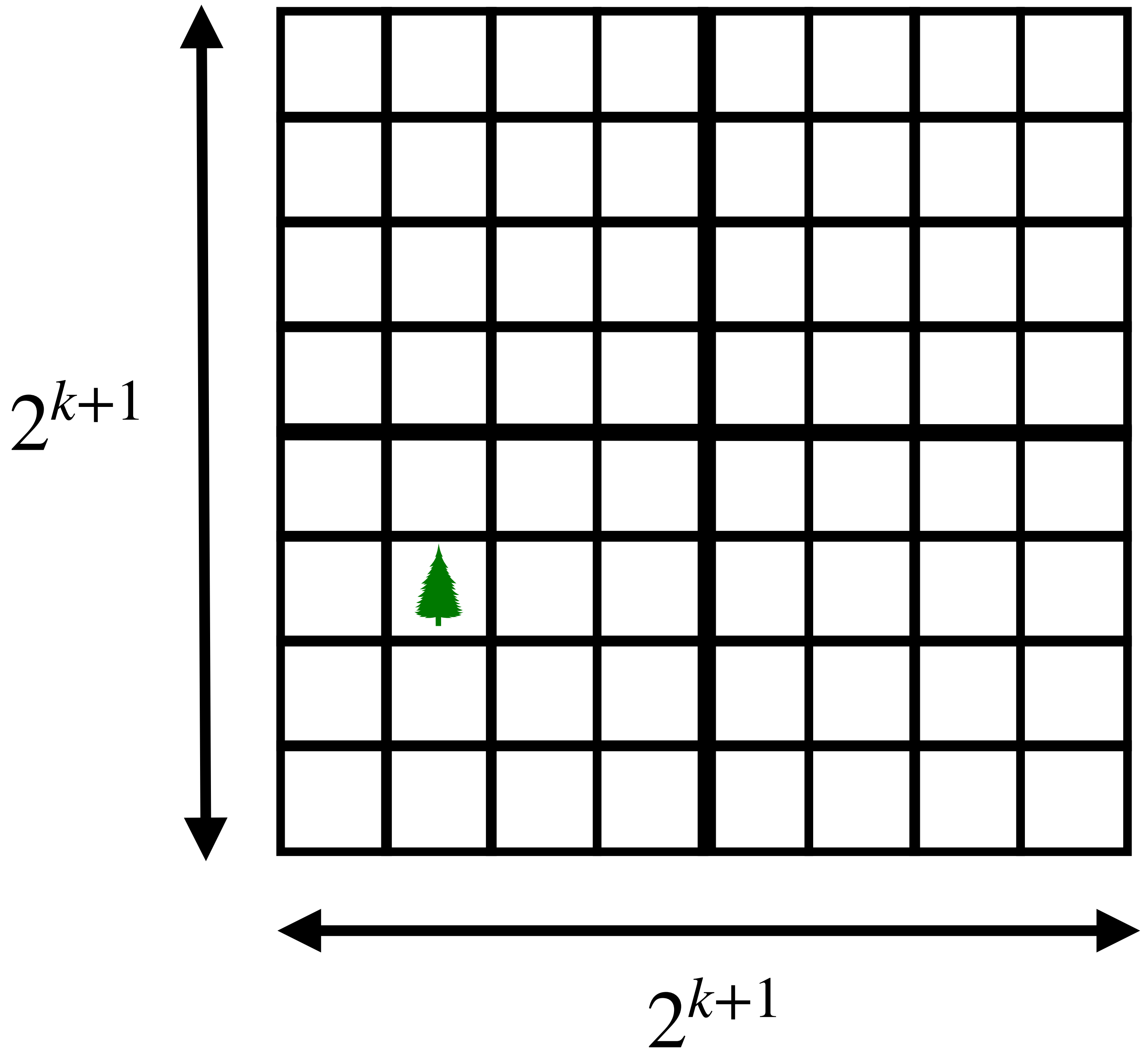

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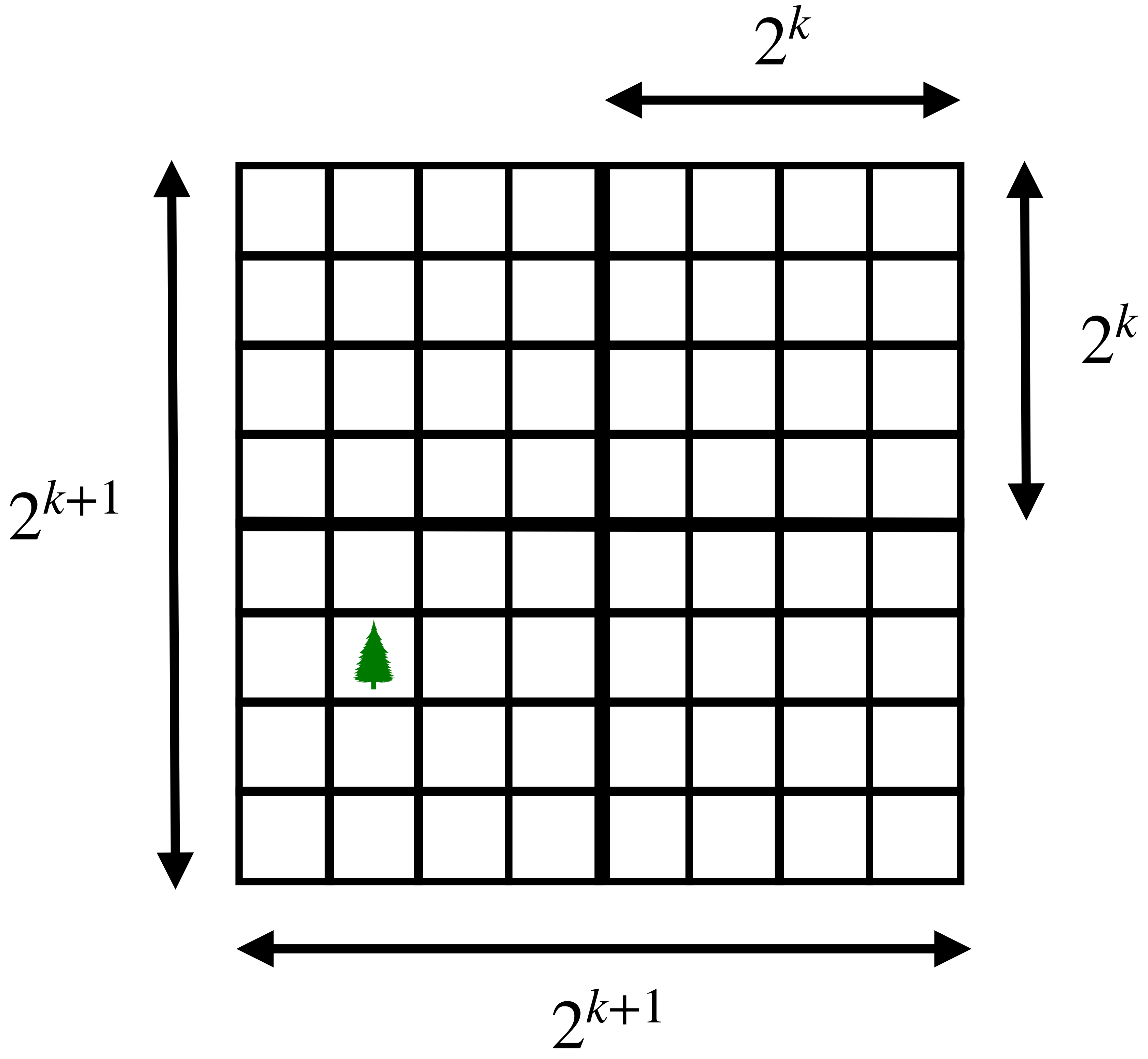

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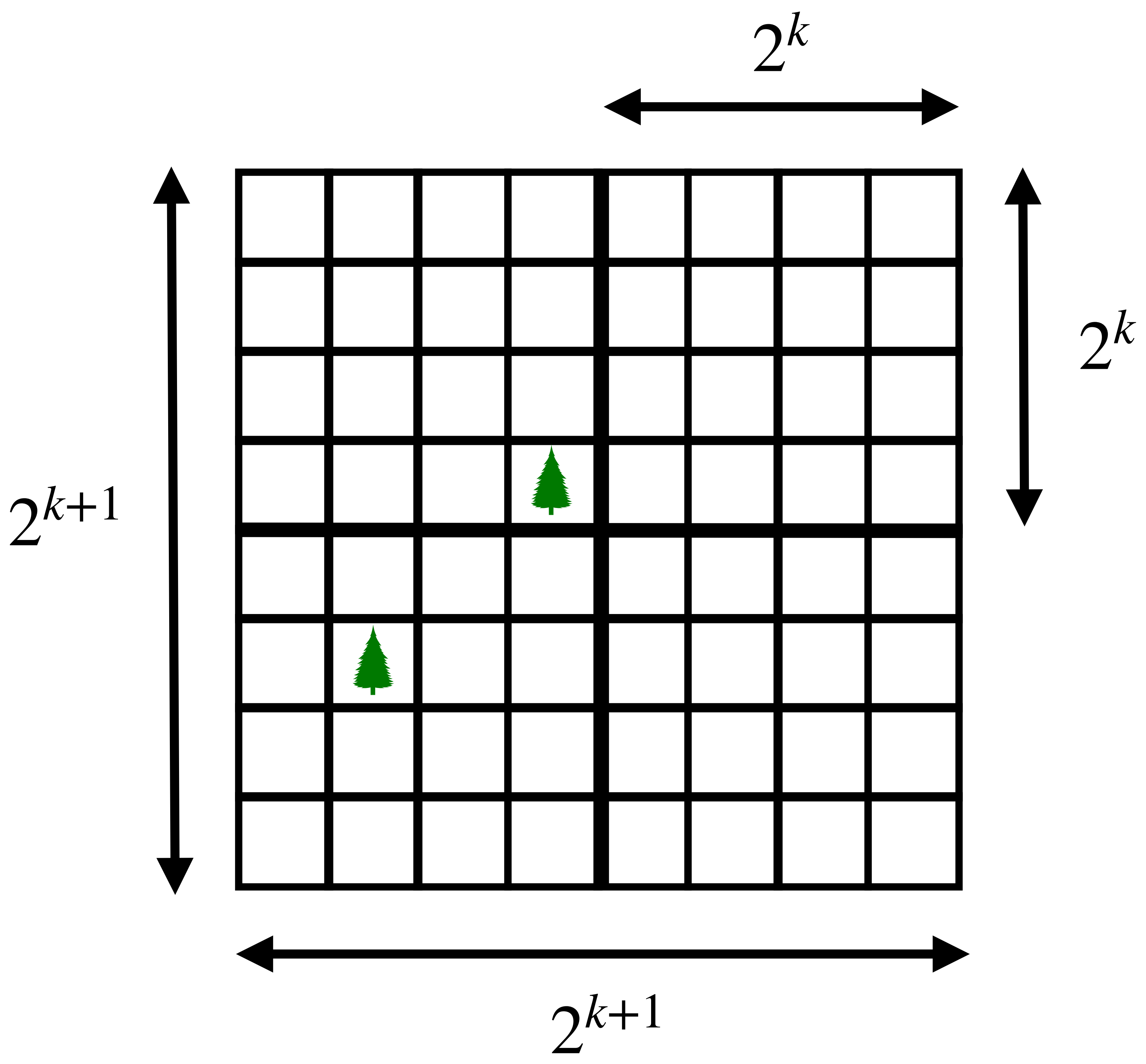

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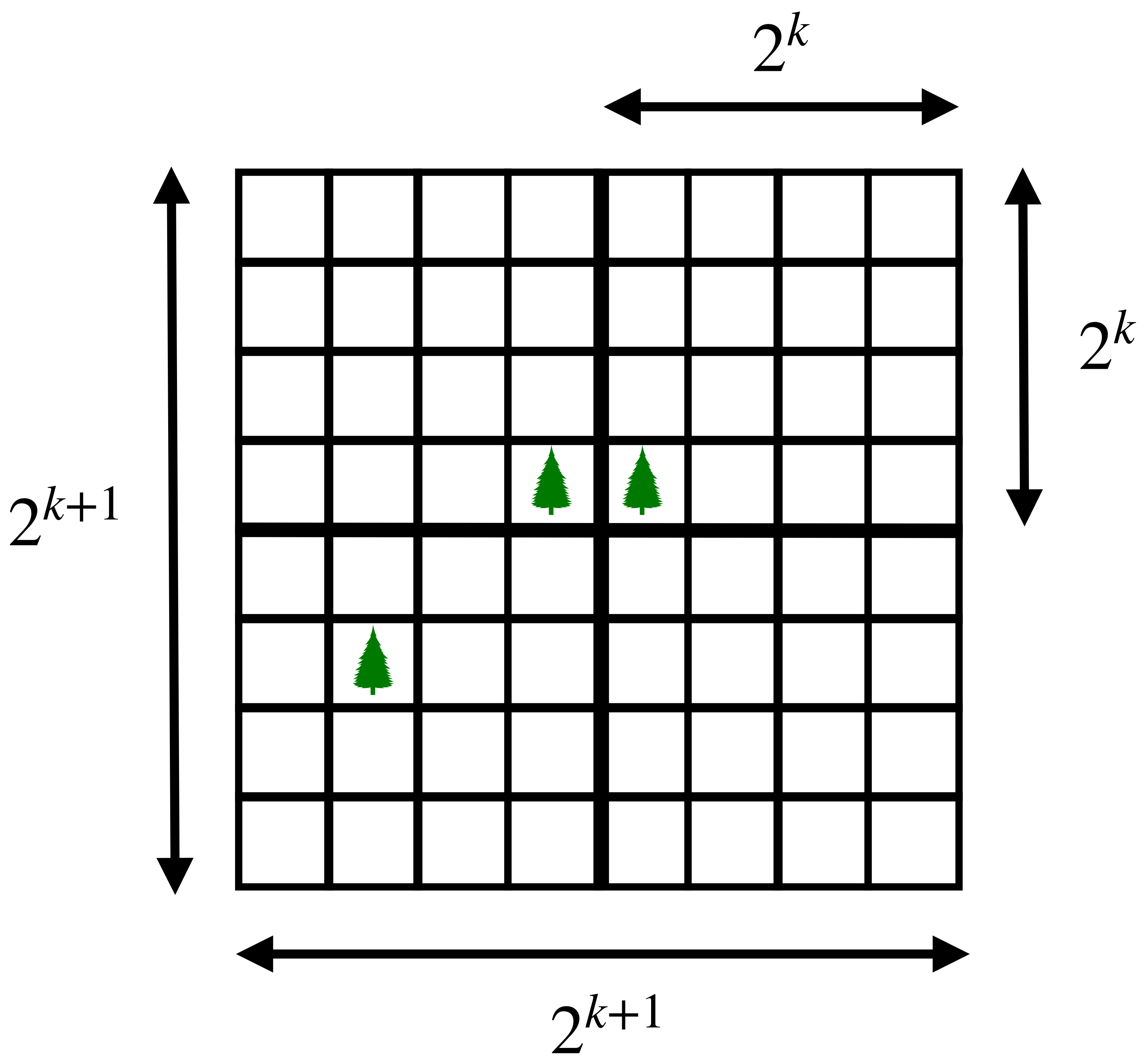

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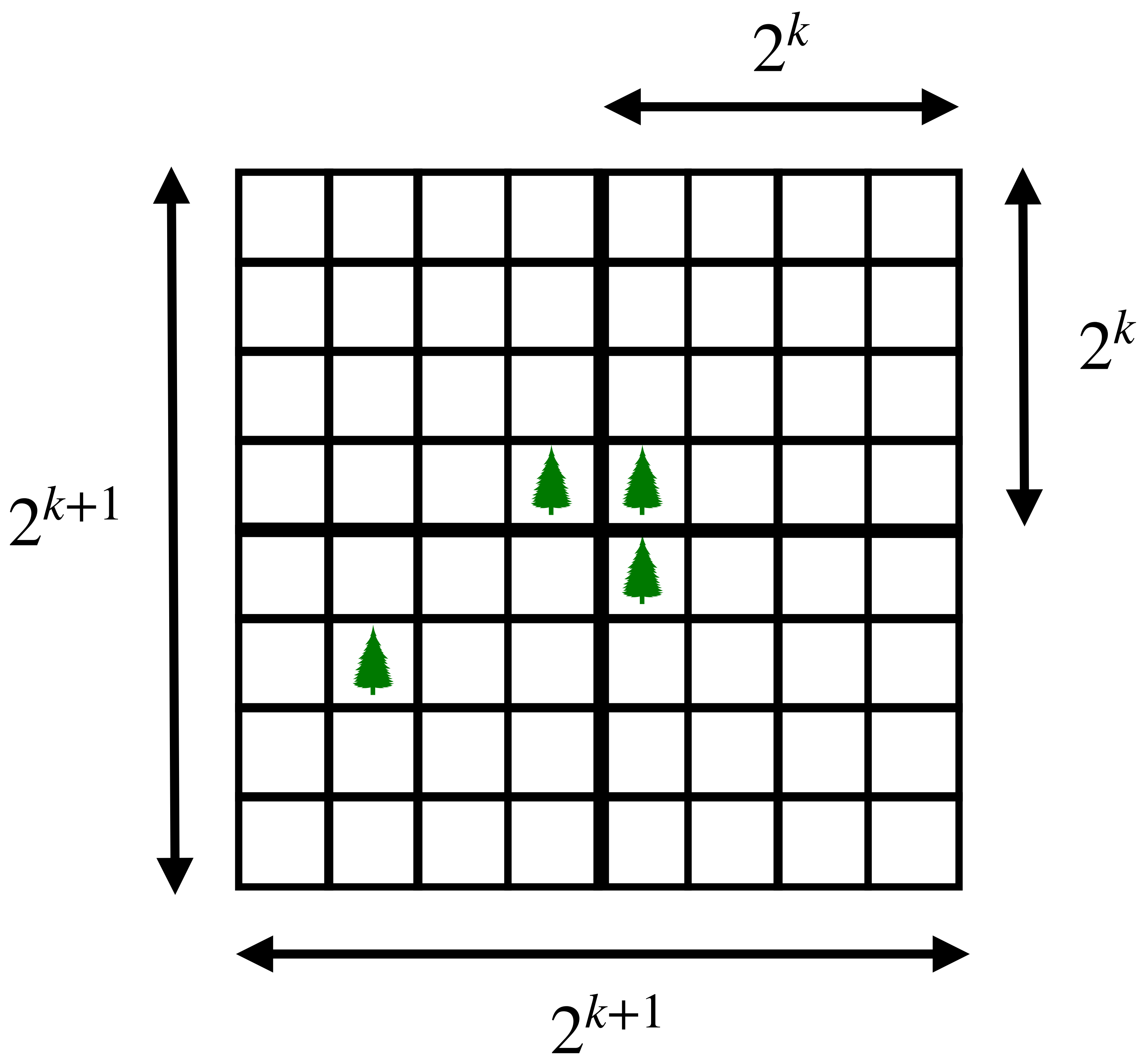

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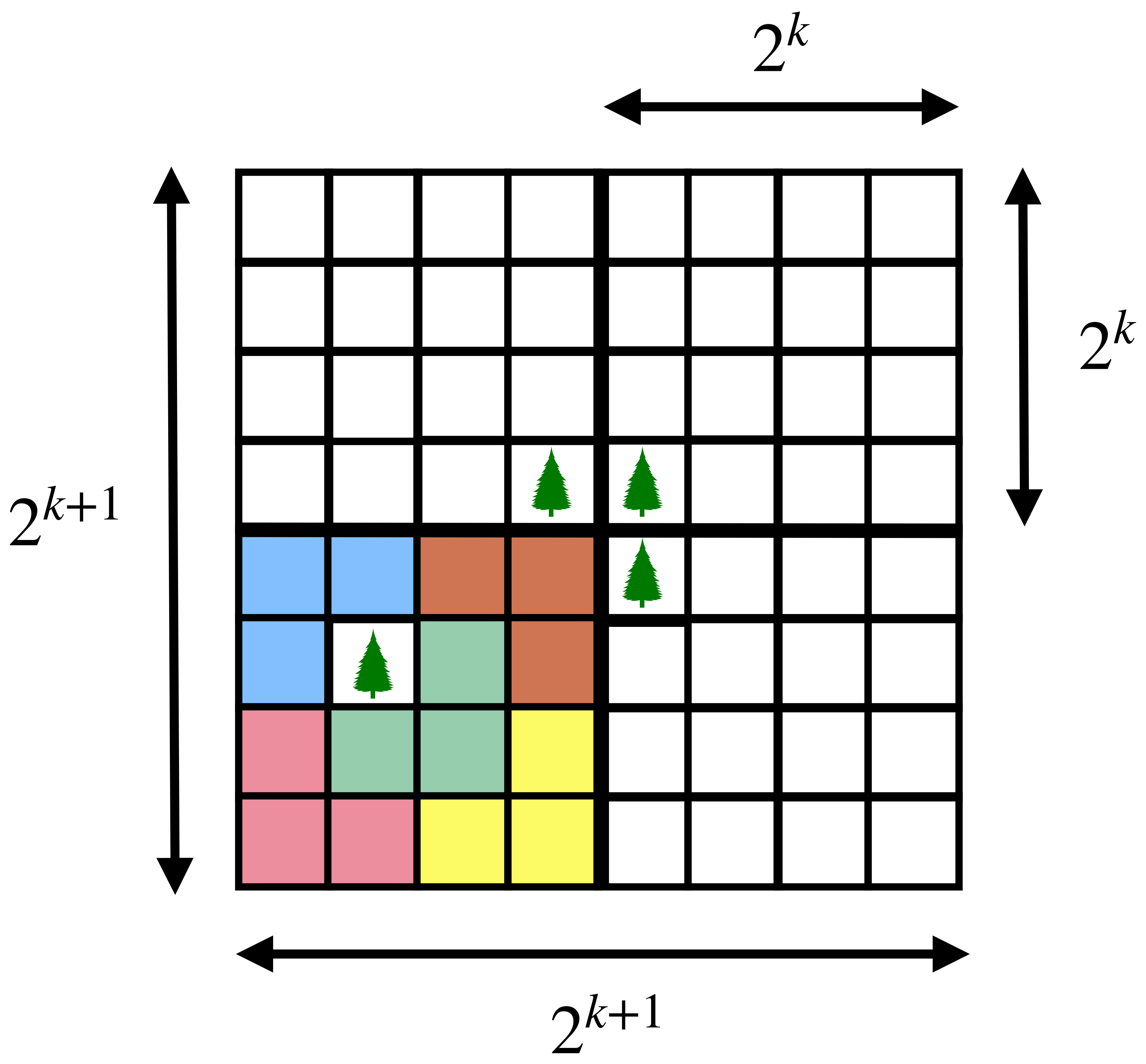

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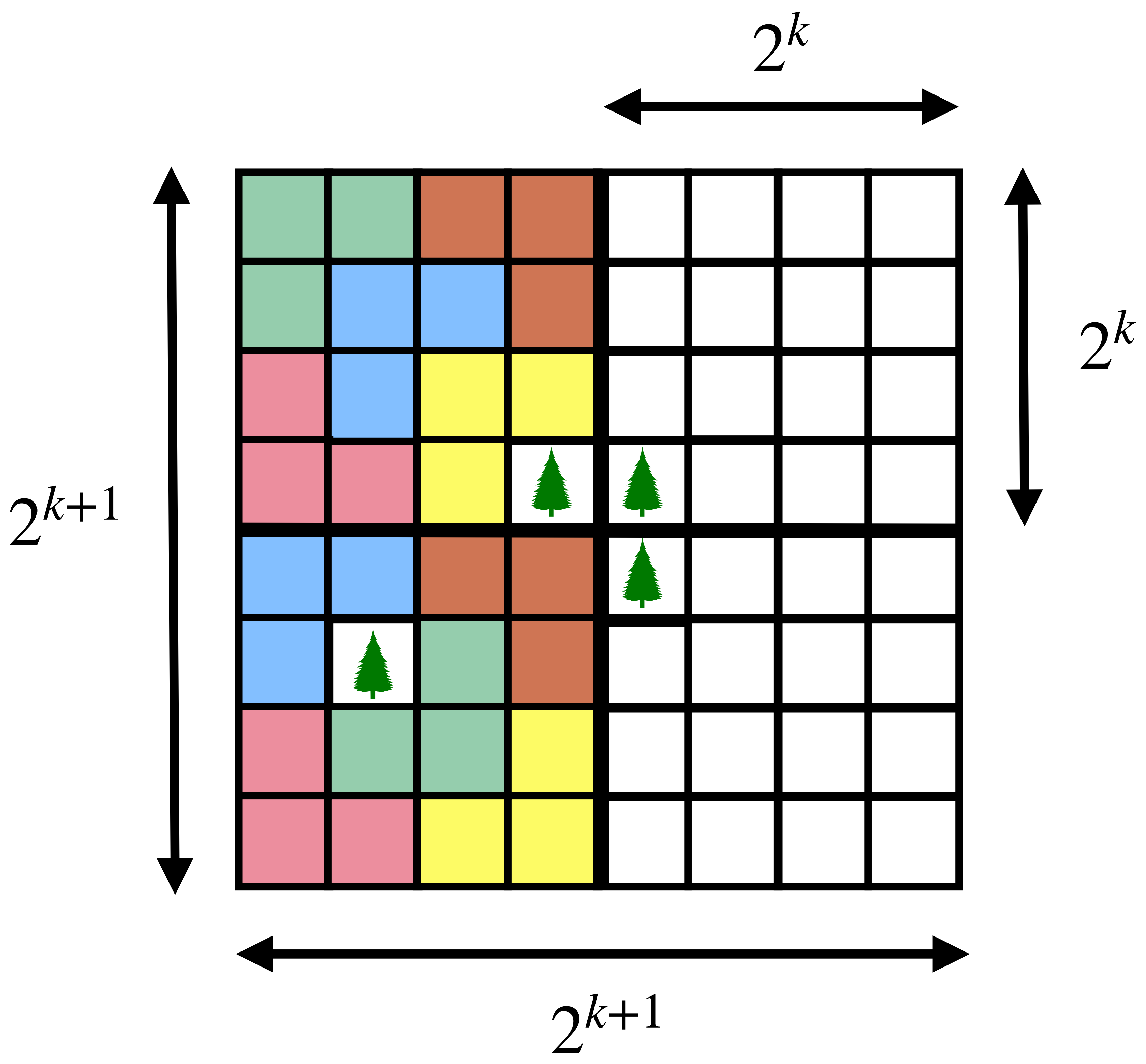

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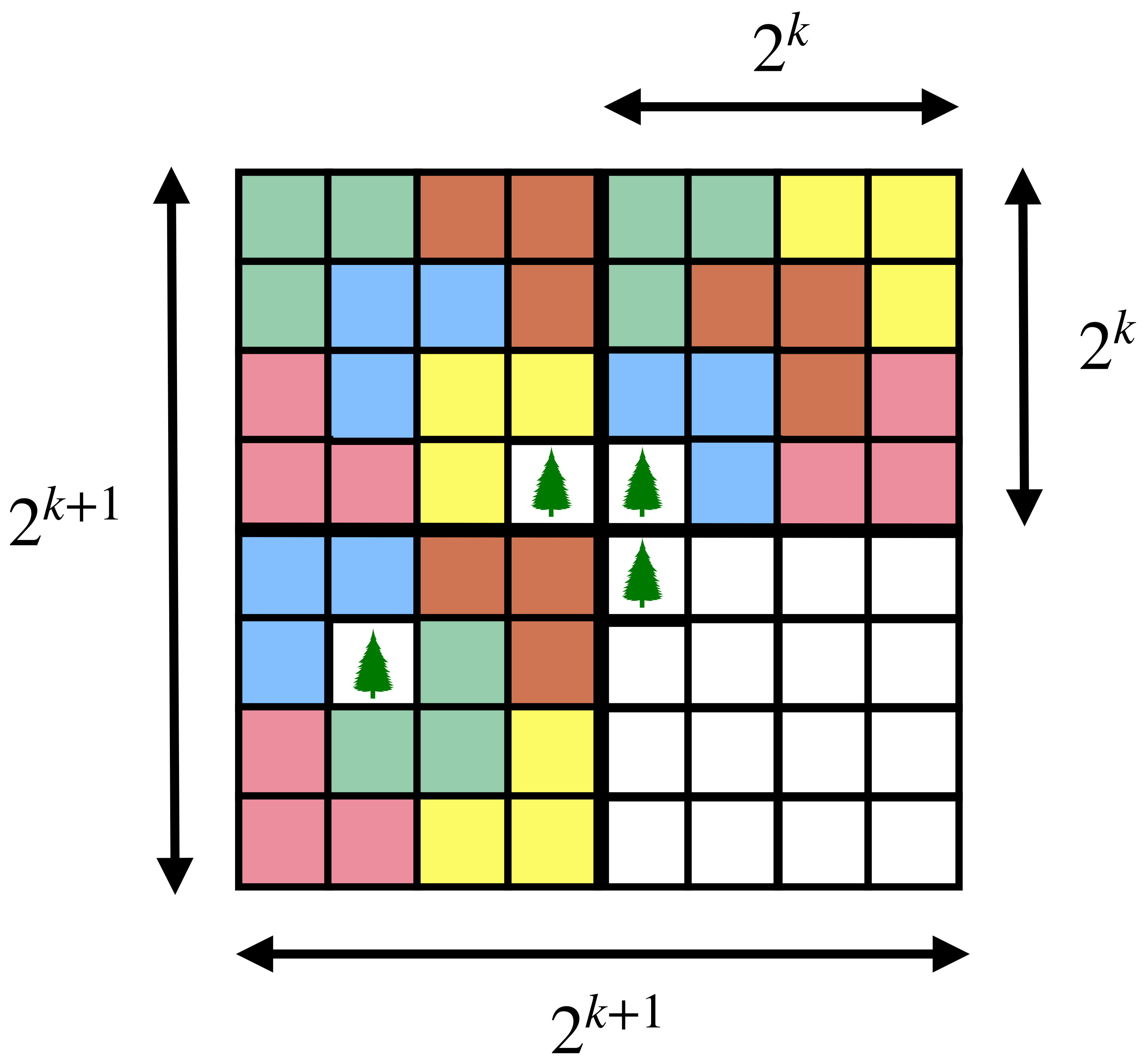

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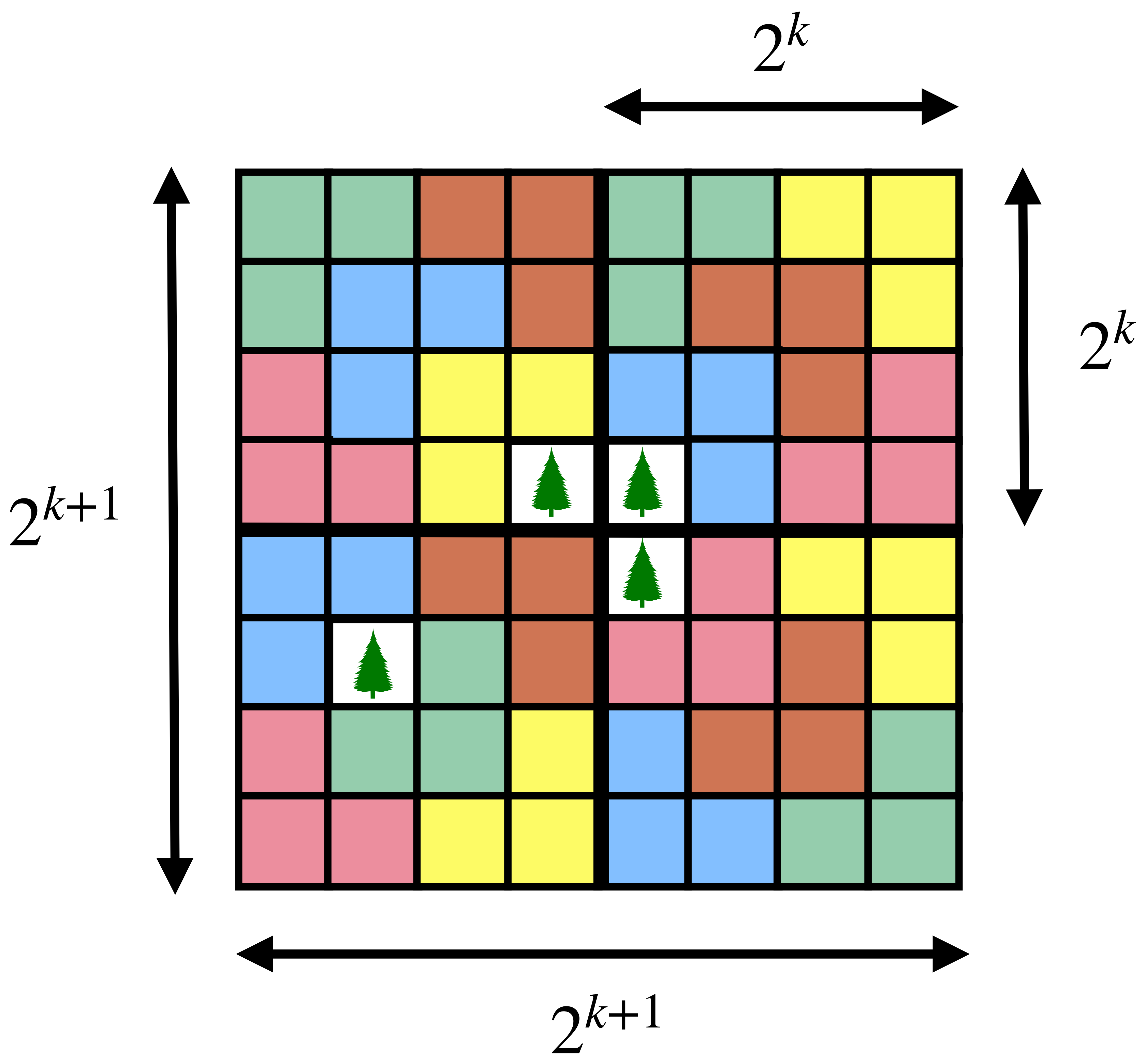

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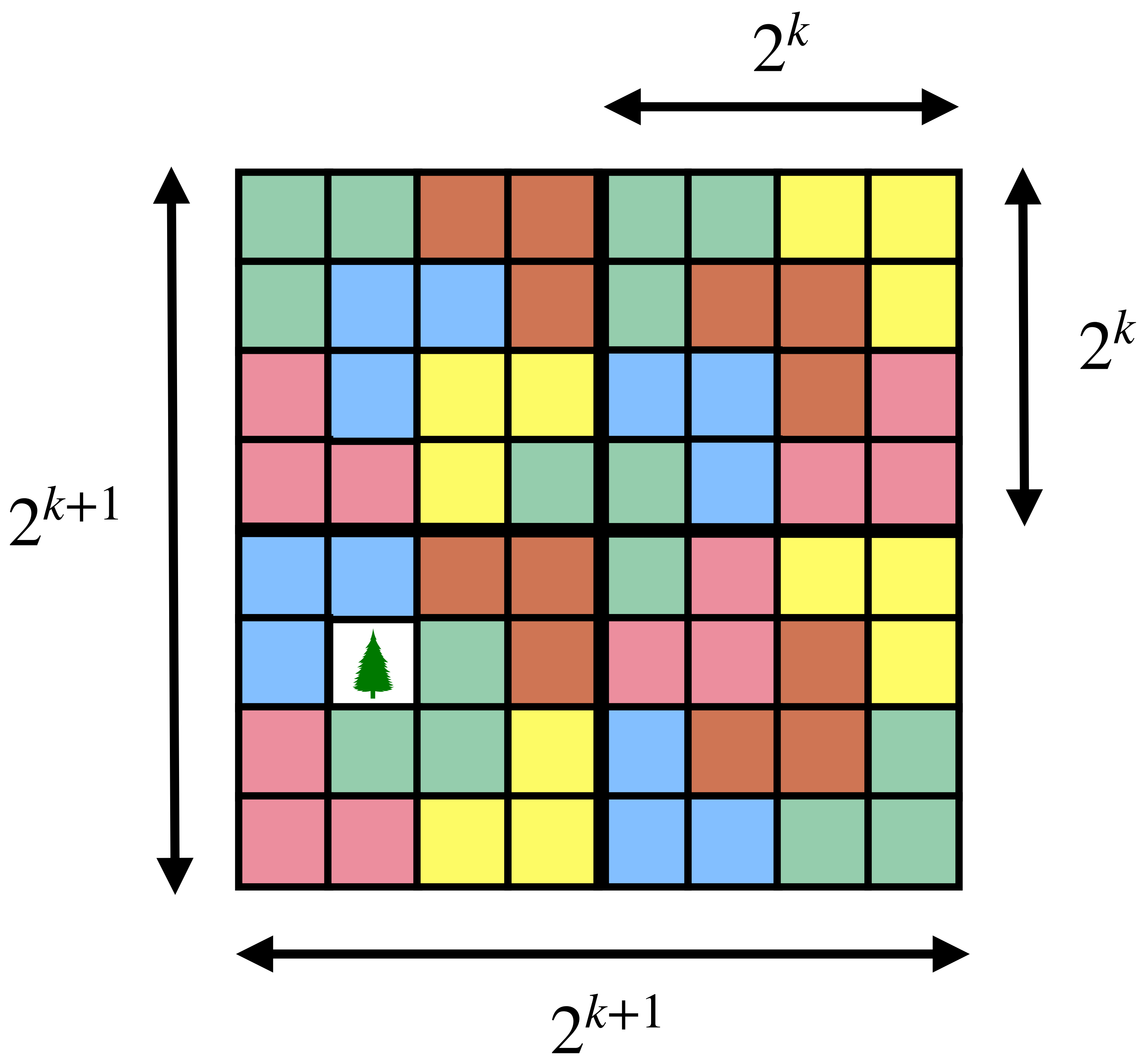

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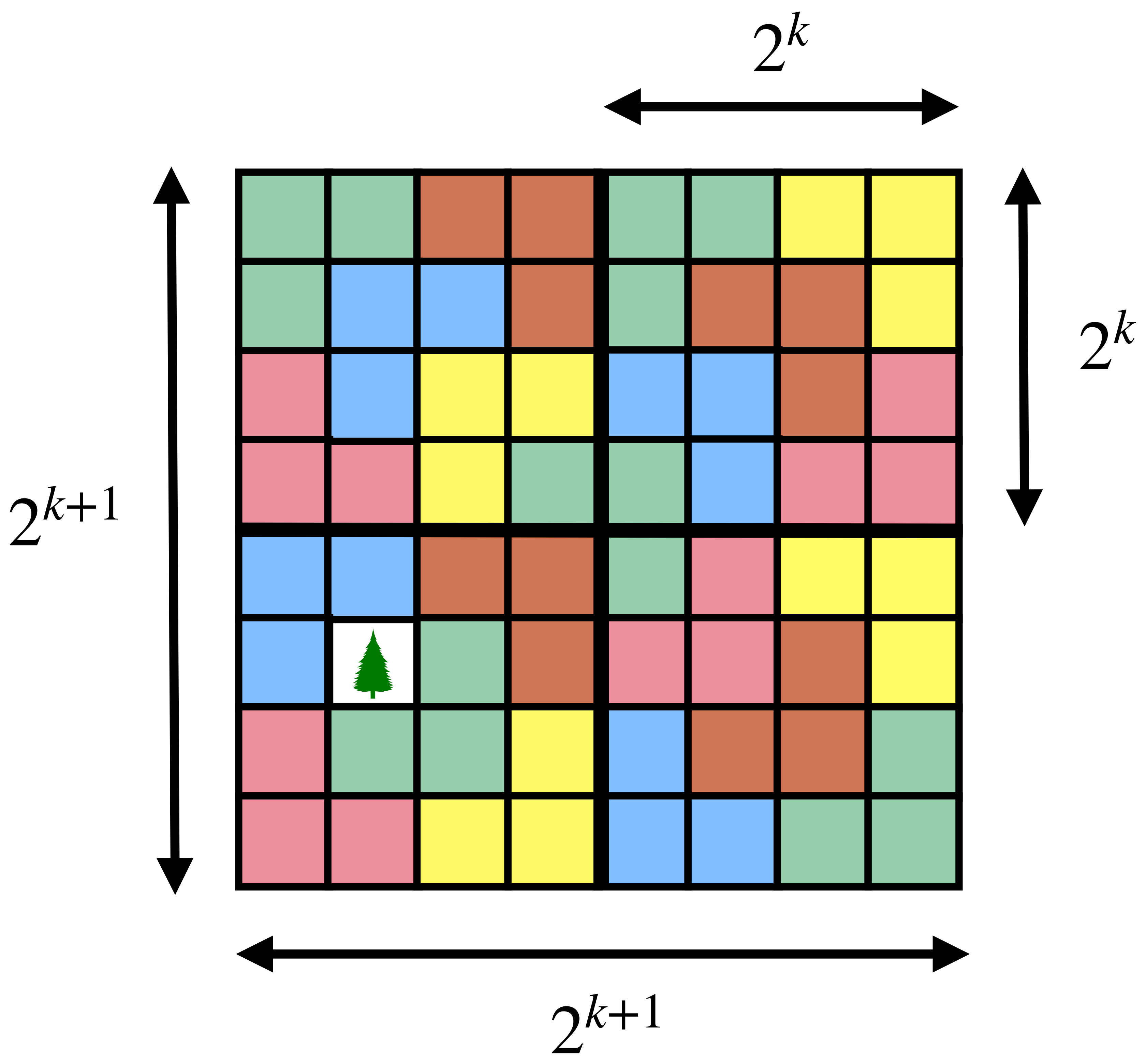

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False Theorem: All positive integers of the form $2n + 1$ are divisible by 2.

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IH implies that $2k + 1 = 2c$.

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Adding 2 on both sides of $2k + 1 = 2c$ gives us $2k + 3 = 2(c + 1)$.

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Under that assumption prove that it is true for $k + 1$ as well, i.e.,

$2k + 3$ is divisible by 2

IH implies that $2k + 1 = 2c$.

Adding 2 on both sides of $2k + 1 = 2c$ gives us $2k + 3 = 2(c + 1)$.

Thus, $2k + 3$ is divisible by 2.

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Inductive Step: Assume that the statement is true for k, i.e,

$$2k + 1 \text{ is divisible by } 2$$

Under that assumption prove that it is true for $k + 1$ as well, i.e.,

$$2k + 3 \text{ is divisible by } 2$$

IH implies that $2k + 1 = 2c$.

Adding 2 on both sides of $2k + 1 = 2c$ gives us $2k + 3 = 2(c + 1)$.

Thus, $2k + 3$ is divisible by 2. $\blacksquare$

Incorrect usage of Mathematical Induction

False Theorem: All positive integers of the form $2n + 1$ are divisible by 2.

Incorrect Proof: We will prove the statement using mathematical induction.

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Error of this proof.

Be careful in base cases.

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1st, 2nd, …, kth horses are of same colour.

If 1st to kth horses have the same colour and 2nd to $(k+1)$th horses have the same colours

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2nd, 3rd, $\ldots$, $(k + 1)$th horses are of same colour.

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1st, 2nd, $\ldots$, kth horses are of same colour.

If 1st to kth horses have the same colour and 2nd to $(k + 1)$th horses have the same colours as well, then all $k + 1$ horses must have the same colour.

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Principle of Mathematical Induction:

To prove $P(n)$ is true for all positive integers n, where $P(n)$ is a propositional function, we perform two steps:

Basis Step: Prove that $P(1)$ is true, unconditionally.

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It did not prove $P(1) \rightarrow P(2)$.

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Plan: We will try to prove assuming very few basic things.

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There are two cases for $k + 1$:

- If $k + 1$ is prime, then we are done.
- If $k + 1$ is not a prime, then $k + 1 = p . q$ for $p, q > 1$.

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Let's learn strong induction!